Signal Analysis & Communication ECE355

Ch. 1·5 : Introduction to Systems

Ch. 1·6 : Properties of System

1. Memorylessness
2. Invertibility.

Lecture 5

18-09-2023

Ch. 1.5: Continuous And Discrete Time Systems

System

- A process that transforms an I/P sig. to an O/P sig.

CT: $x_{(t)} \longrightarrow \boxed{T} \longrightarrow y_{(t)}$ $\quad y_{(t)} = T\{x_{(t)}\}$

CT Sys.

DT: $x_{[n]} \longrightarrow \boxed{T} \longrightarrow y_{[n]}$ $\quad y_{[n]} = T\{x_{[n]}\}$

DT Sys.

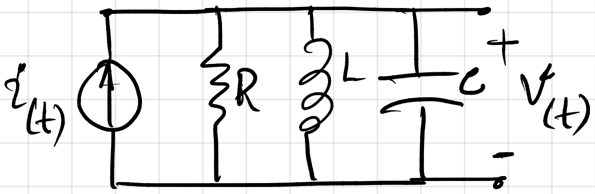
Characterized by I/p–o/p relationship

- Examples

① Automobile: I/p – force $\longrightarrow \boxed{T} \longrightarrow$ O/p – velocity of the vehicle

② Image Enhancement Sys: I/p – image $\longrightarrow \boxed{T} \longrightarrow$ O/p – image with improved contrast

③ RLC Sys.:

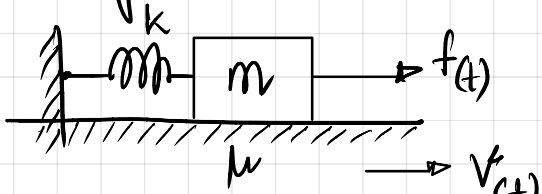


I/p–O/p relation

$$i_{(t)} = \frac{V_{(t)}}{R} + \frac{1}{L}\int_0^t V_{(\tau)}\,d\tau + C\frac{dV_{(t)}}{dt}$$

I/p – $i_{(t)}$
O/p – $V_{(t)}$

④ Spring-mass Sys.



$f_{(t)}$: force
m: mass
k: spring constant
μ: friction coeff.
$V_{(t)}$: velocity

I/p–O/p relation

$$f_{(t)} = \mu V_{(t)} + k\int_{-\infty}^{t} V_{(\tau)}\,d\tau + m\frac{dV}{dt}$$

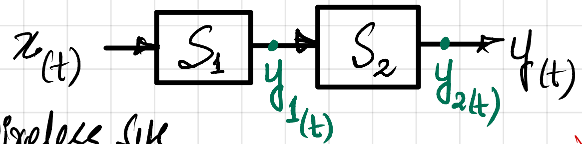
I/p – $f_{(t)}$
O/p – $V_{(t)}$

* Mathematical Description of Sys. from wide variety of applications frequently have a great deal in common

Interconnection of Systems

- Many real systems are built as interconnection of several subsystems.
- By describing a sys. in terms of an interconnection of simpler subsystems, we are able to define useful ways to synthesize complex systems.
- Basic interconnections:

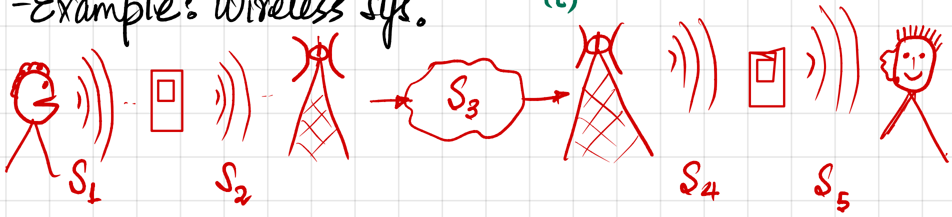
① Series



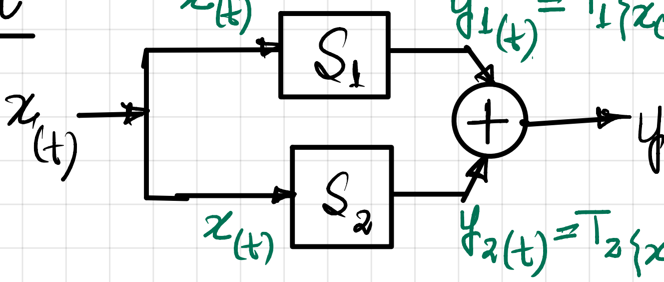
$y_1(t) = T_1\{x(t)\}$

$y(t) = y_2(t) = T_2\{y_1(t)\} = T_2\{T_1\{x(t)\}\}$
$= T\{x(t)\}$

- Example: Wireless sys.



② Parallel

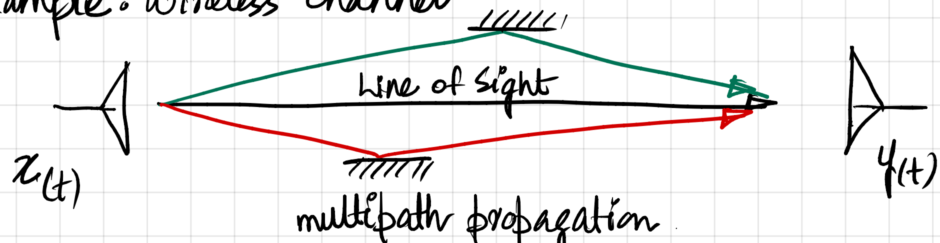


$y_1(t) = T_1\{x(t)\}$

$y(t) = T_1\{x(t)\} + T_2\{x(t)\} = T\{x(t)\}$

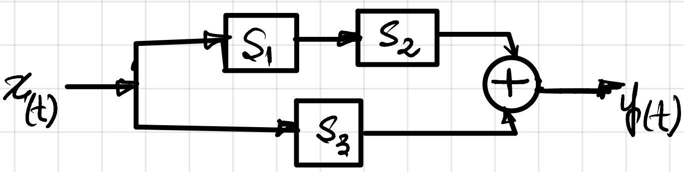
$y_2(t) = T_2\{x(t)\}$

- Example: Wireless Channel

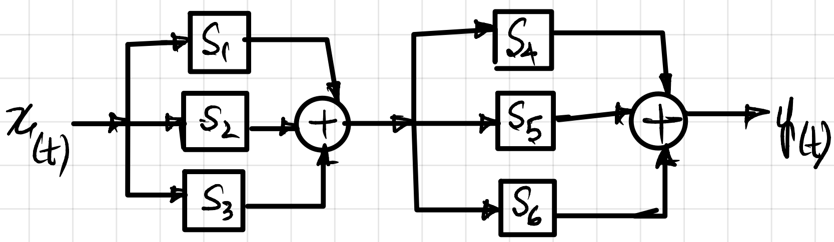


③ Series Parallel

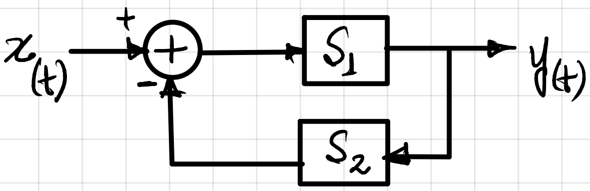
- Combination of the above two



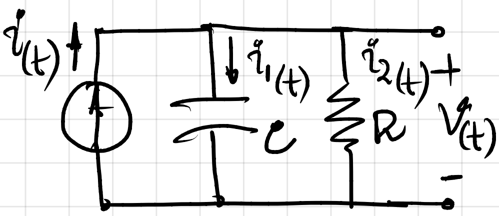
OR



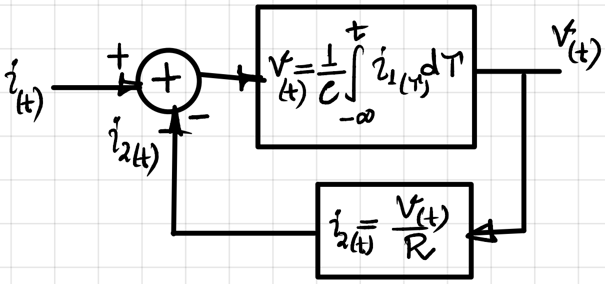
④ Feedback



- Example:
  1. A cruise control sys. senses the vehicle's velocity & adjusts the fuel flow to keep the speed at the desired level.
  2. RLC Circuit



$$i_{1(t)} = i_{(t)} - i_{2(t)}$$



Ch. 1·6: BASIC SYSTEM PROPERTIES

- Have important physical interpretation & simple mathematical descriptions
  → useful for understanding / applying sig. & sys. concepts.

① Systems With & Without Memory

Ⓐ Memoryless

Defn: A system is memoryless if for all times $t \in \mathbb{R} / n \in \mathbb{Z}$, the output value $y_{(t)} / y_{[n]}$ depends only on the input value $x_{(t)} / x_{[n]}$ at that same time.

- meaning → only the current time matters, not the past or future.
- Examples: ① the squaring system: $y_{(t)} = x^2_{(t)}$
  ② the modulation system: $y_{(t)} = A\sin(\omega_0 t) \cdot x_{(t)}$ (study later)
  ③ $y_{[n]} = (2x_{[n]} - x^2_{[n]})^2$
  ④ the Resistor: $V_{(t)} = R\, i_{(t)}$
  ⑤ the identity system: $y_{(t)} = x_{(t)}$
  $y_{[n]} = x_{[n]}$

Ⓑ With Memory

- Presence of a mechanism in the sys. that retains or stores information about input values at non-current times.
- Example: ① the Accumulator sys. $y_{[n]} = \sum_{k=-\infty}^{n} x_{[k]}$
  ②* the time delay sys. $y_{(t)} = x_{(t-1)}$
  ③ the derivative sys. $y_{(t)} = dx_{(t)}/dt$ | $y_{(t)} = \lim_{h \to 0} \frac{x_{(t+h/2)} - x_{(t-h/2)}}{h}$
  ④ the capacitor: $V_{(t)} = \frac{1}{C}\int_{-\infty}^{t} i_{(\tau)} d\tau$.
  ⑤ Computers: Storage registers (memory)

* Some cases where current O/P is dependent on future I/P values (e.g. $y_{(t)} = x_{(t+1)}$)
We'll see later.

② Invertibility

Defn: A system 'T' is said to be invertible if there exists another system 'T_{inv}' such that

$$\left.\begin{aligned} T_{inv}\{T\{x_{(t)}\}\} &= x_{(t)} \\ T_{inv}\{T\{x_{[n]}\}\} &= x_{[n]} \end{aligned}\right\} \text{ for all time } t \in \mathbb{R} / n \in \mathbb{Z}$$

- Meaning → you can "undo" the operation of T.

Examples: ① $y_{(t)} = 2x_{(t)}$

Inverse $w_{(t)} = \frac{1}{2} y_{(t)} = x_{(t)}$ invertible

② the delay system, $y_{(t)} = x_{(t-T)}$

$w_{(t)} = y_{(t+T)} = x_{(t)}$ invertible

③ the squaring sys., $y_{(t)} = x^2_{(t)}$

$w_{(t)} = \pm\sqrt{y_{(t)}}$ Not invertible

(as here, $+\sqrt{y_{(t)}}$ & $-\sqrt{y_{(t)}}$ both results in $y_{(t)}$; $x_{(t)}$ could be $+\sqrt{y_{(t)}}$ or $-\sqrt{y_{(t)}}$)

* Another defn: A sys. is said to be invertible if distinct I/Ps leads to distinct O/Ps.

④ $y_{(t)} = 0$ Not invertible

(For any I/P, O/P is zero)

⑤ Accumulator: $y_{[n]} = \sum_{n=-\infty}^{n} x_{[n]}$

$$= \sum_{n=-\infty}^{n-1} x_{[n-1]} + x_{[n]}$$

$$= y_{[n-1]} + x_{[n]}$$

$$\Rightarrow x_{[n]} = y_{[n]} - y_{[n-1]}$$ invertible

⑥ Encoding in Dig. Comm. (Compression, Error Control, Encryption)

all invertible

$$x_{[n]} \to \boxed{S} \xrightarrow{y_{[n]}} \boxed{S^{-1}} \to w_{[n]} = x_{[n]}$$